Putting Chaos to Work

Chaos is usually something to avoid, but it might come in handy for mixing industrial composites as quickly and efficiently as possible

JULIO OTTINO PRODUCES STRIKING IMAGES. Bright ribbons of red and yellow twirl across a dark background, dividing and subdividing while weaving around one another like dancers in a cotillion. A single splotch of color stretches and shudders until it disintegrates into a Jackson Pollock–like spray of thousands of tiny dots. Animated cartoons? Modern art? No, it's modern science.

With his co-workers in the chemical engineering department at the University of Massachusetts at Amherst, Ottino studies how fluids mix. He injects dyes into clear fluids and watches as they are stirred and blended by the motion of his mixing apparatus. The colorful sequences of still photographs produced by these studies are, Ottino says, offering insights into the fundamentals of fluid mixing, a field that has previously had little theoretical grounding. In particular, he says, he has learned that the key to efficient mixing of fluids lies in chaos—the ordered complexity that has been one of science's "hot topics" over the past few years.

Ottino's work has caught the eye of a number of basic scientists, from mathematicians investigating chaos to geophysicists studying the earth's mantle. And now it is beginning to attract attention among a much more applied group: the chemical engineers who design industrial mixing devices. "We are firmly convinced that we have to understand mixing [theoretically]," says Costas Gogos of the Polymer Processing Institute at Stevens Institute of Technology in Hoboken, New Jersey. Almost every industrial process, from mining and chemical refining to polymer production and waste water treatment, involves mixing at some stage.

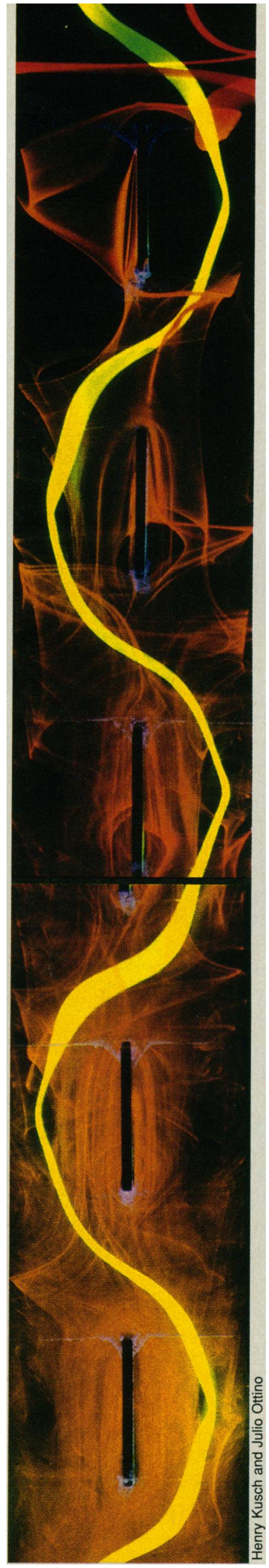
If Ottino is right, engineers may be able to get better mixing and improve the efficiency of many of these operations by learning how to design for and direct the chaos that often appears naturally in the systems.

Some mixing is easy—the "turbulent mixing" of cream stirred into coffee, for instance. Although turbulent mixing is not well understood mathematically, it's efficient and straightforward to obtain as long as the fluids flow easily.

But when one or both of the fluids are viscous, as when two thick paints are stirred together or water is folded into a stiff bread dough, it's another matter. Such materials won't mix turbulently, says Arthur Metzger, a chemical engineer at the University of Delaware in Newark, so "you try to layer them in a helpful way." Years of trial and error have taught engineers how to make machines that do a relatively good job of mixing viscous materials, he says, but there is no basic theory of mixing that explains why one device works and another doesn't.

That's where Ottino comes in. For a decade, he has been working on a mathematical description of how viscous fluids interact. He is "laying the theoretical groundwork for the analysis of mixing," says Chris Rauwendaal, a chemical engineering consultant in Los Altos Hills, California. The systems Ottino works with are much simpler than those in industry, as he is the first to admit, but that simplicity has allowed him to identify some basic features of fluid mixing that had previously been lost in the complexity of the flows. The pictures—some of the most beautiful images of chaos produced to date—are an added bonus.

One of Ottino's uncomplicated mixers, built with graduate student Mahari Tjahjadi, consists of a squat, hollow cylinder with a smaller tube inside. The assembly is sealed on both ends and viscous fluid is inserted between the two tubes, which can rotate independently of each other. When the cylinders turn, they drag the fluid along with them, mixing it as a result. The researchers trace the mixing action by marking a small region of the fluid with an organic dye and photographing the system as it evolves. The system is essentially two dimensional, Ottino says, because the dye remains within a single layer during the mixing. Despite the device's simplicity, it can manifest some surprisingly complex behavior (see photo on page 627).


Henry Kusch and Julio Ottino

Order and chaos. *Depending on its placement in the tube, dye is either well mixed or almost undisturbed.*

Indeed, it was in this system that Ottino first demonstrated the presence of chaotic mixing. If the cylinders are set to rotate in certain patterns, they create a "stretching and folding" action, in which regions of the fluid are elongated and then doubled back on themselves, much like taffy in a taffy pull machine or bread being kneaded. As was shown by mathematician Stephen Smale of the University of California at Berkeley, this pattern of stretching and folding leads to chaos. Since that implies that two nearby points in the fluid move away from each other exponentially fast, such chaotic mixing is the most efficient way to mix viscous fluids quickly, Ottino says.

But the presence of chaos in the mixer is not enough to guarantee that all its contents will be well mixed. While some of the regions in the fluid may be mixed chaotically, Ottino says, other parts may be nearly unaffected, and he has eye-catching photographic proof. In one series of photos, he puts two dabs of dye into the cylindrical mixing system. One is placed in a chaotic mixing region, where it stretches and doubles back on itself until it finally breaks into little pieces. But the second, located in an "island of stability," mostly holds its shape as it moves around the mixing chamber.

Ottino has found similar behavior in more complicated three-dimensional mixing systems, including one resembling a type of mixer used industrially. He and student Henry Kusch built a long vertical tube with a series of short partitions that divide each segment of the tube in two. As glycerine is passed through the device, the researchers rotate the tube at various speeds while keeping the partitions fixed. The flow of the fluid through this "partitioned pipe mixer" is similar to that through a common commercial mixer called a static mixer. Once again, the dye markers show that some parts of the fluid are chaotically mixed while others are practically untouched. These unmixed regions appear in photographs as long colored ribbons that stretch through the mixer.

Such stable areas have to be eliminated if a fluid is to be well mixed, and the mathematical theory that Ottino has been developing alongside his experimental work provides some insights that may help. "We're still trying to understand three-dimensional flows," he says, "but the picture is now pretty clear for two-dimensional, time-periodic flows."

When a two-dimensional mixer repeats the same pattern of motion over and over again, Ottino has found that the mixed fluid will have its own symmetries, producing "order within chaos." The order appears in the form of "periodic points," defined as points in the fluid that keep returning to the same spot in the mixer. If a two-dimensional, time-periodic flow is chaotic, Ottino says, "you are guaranteed periodic points of all orders"—points that return to the same spot after every mixing period, after every second period, after every third period, and so on.

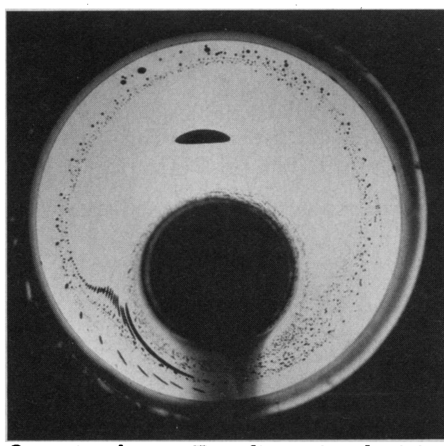
Some of these periodic points lie at the center of the islands of stability that appear in the laboratory mixing experiments, so it is important to be able to predict where they will appear. It's possible to do this mathematically for very simple mixing systems, Ottino says, but the calculations quickly become too involved even for supercomputers once the mixing system becomes complicated. Ottino is now trying to find ways of predicting how fluids will behave "without actually doing the brute force calculations."

But even in the absence of methods for predicting where the islands of stability will form, Ottino has been able to use insights from his mathematical models to suggest some ways to break them up. By modifying the timing of the mixer, for instance, islands of stability can be moved into regions of chaotic mixing. Alternatively, the spatial symmetry of the systems can be broken up slightly to avoid the formation of the stable regions.

It's likely that industrial mixers actually do some of the same things that Ottino does in his lab, Rauwendaal says, but no one knows for sure what goes on inside them. "You probably get [chaotic mixing] now, but our understanding of it is so limited that we don't know when we get it." Ottino's findings may teach engineers how to consciously incorporate chaos into their mixers and use it to best effect.

The value of Ottino's work, says Ronald Rosensweig of Exxon Research and Engineering Company in Annandale, New Jersey, is that it allows researchers to see what is actually happening in simple mixing systems instead of treating them as black boxes. "In a lot of the old work," he says, "people used point probes [which measure fluid velocity at a single point only]. They couldn't get the big picture." Ottino's pictures of fluid mixing in two and three dimensions let researchers develop an intuition for how mixing will proceed in more complicated devices.

Most chemical engineers, however, have not latched onto Ottino's work. Why not? One reason may be that they are uncomfortable with the terms and concepts developed by Ottino. For instance, fluid flow is usually analyzed in terms of its velocity fields—the speed and direction of the fluid at each spot—but Ottino instead traces how individual points in the fluid move around the mixing system. "The mathematical description he uses is not one comfortable to people in the field," Rosensweig says. "Probably only one in 100 [chemical engineers] can really understand what Ottino is doing now."


Mahari Tjahjadi and Julio Ottino

Gone to pieces. *Two-dimensional mixing breaks up one spot of dye while leaving a second spot whole.*

But if chemical engineers are slow to apply what Ottino does to their own work, the same is not true for researchers in some less applied fields. At the University of Maryland, for example, Edward Ott has mathematically analyzed Ottino-type chaotic mixing. "If you do what Ottino does, you get all these striations where the dye density is changing rapidly" from one place to the next, Ott notes. "What is the character of these regions that have the very sharp changes in density?" This question can be made mathematically rigorous, and Ott has a mathematical answer: "They tend to become fractal." Fractals are intricate objects that are self-similar—that is, they look the same when they are magnified 10 or 1000 or 1 million times—and they pop up in connection with chaos. In this case, Ott says, as a fluid containing one region of dye is stretched and folded over and over again, the areas in which the concentration of the dye changes sharply become concentrated into a fractal-shaped region of the fluid.

And at Cornell University in Ithaca, New York, geophysicist Donald Turcotte believes that chaotic mixing may even proceed in the earth's mantle—albeit much more slowly than in Ottino's experiments. In Turcotte's model of the mantle, 6-kilometer-thick basalt plates plunge into the earth's interior at subduction zones in the middle of the ocean and are stretched and mixed with other matter before they are cycled up to the surface again, hundreds of millions of years later.

Turcotte came to this conclusion from his examination of certain rocks that have centimeter-thick layers of basalt that he says are the result of the kilometers-thick basalt being stretched incredibly thin. At the temperatures and pressures present in the earth's mantle, even basalt flows like a liquid—a very viscous liquid, he notes. Pictures of the rocks are very similar to Ottino's pictures of chaotically mixed fluids, and Turcotte sees this as evidence for stretching and folding in the earth's mantle.

But Ottino, who was trained as a chemical engineer, would really like to see some practical applications of his work. "In the past, people have implicitly used chaos in [commercial] mixing," he says. "Once you have the tools and can identify chaos, you may be able to improve upon it." In other words, he would like to throw a little chaos into his own field, but not throw his field into chaos.

■ ROBERT POOL

ADDITIONAL READING

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